

Effect of thermophoresis and heat sources on hydro magnetic convective heat and mass transfer flow past a vertical wavy wall with variable viscosity, thermal conductivity, thermal radiation and Chemical reaction

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I. INTRODUCTION:

The study of theremophoresis plays a vital role in the species transport mechanism of several devices consists of small micron sized particles and large temperature gradient. In fact, the particles are moving from hot surfaces to cold one. As small such particles (example dust) are suspended in a gas with a temperature gradient, experience a force in the direction opposite to the temperature gradient. The effect of the thermophoresis is so widespread in many practical applications in removing small particles from gas streams, in studying particles material deposition on turbine blades and in determining exhaust gas particle trajectories from combustion devices. This shows that thermophoresis is the dominant mass transfer mechanism in the modified chemical vapor deposition (MCVD) process as currently used in the fabrication of optical fiber performs. Thermophoretic deposition of radioactive particles is considered to be one of the important factors causing accidents in nuclear reactors

Several authors, Duwairi and Damseh et al [10], Damseh et al [7], Mahdy and Hady [21], Liu et al [20], Postelnicu [25], Dinesh and Jayaraj [9], Grosan et al [13], Tsai and Huang [34] have investigated the effect of thermophoresis in vertical plate, micro-channel, horizontal plate and parallel plate.

In light of these applications, the effect of thermophoresis in laminar flow of a viscous incompressible fluid was first established by Goren(12). We and Grief(36) studied the thermophoresis deposits including an application to the outside vapour deposits process. Talbolt et al(33) analysed thermophoresis of particles in a heated boundary layer.

In recent years, considering the influence of thermophoresis ,investigations on the free or mixed convection flow of viscous incompressible fluid past or along surface, vertical and horizontal surfaces, stretching sheet, wedge, cylinder and sphere under different boundary conditions have been accomplished by, Duwairi and Damseh et al [10], Damseh et al [7], Grosan et al(13), Mahdy and Hady [21]. Liu et al [20]. Javarai et al(17). Selim et al(29).Seddeek(28).Alam et al(1).Partha(8).Chen and Chan(24). Puvi Arasu et al(26),Das(8),Postelnicu(25) ,Sreenivasa Reddy et el(31,32), Ganesan et al(11).and Aliveni et al(2). But Wang and Chen(35) was the first investigated thermophoresis deposition of particles from a boundary layer flow onto a continuously moving wavy surface without considering the Darcy porous medium.In view of theses applications.

With the fuel crisis deepening all over the world, there is a great concern to utilize the enormous power beneath the earth's crust in the geothermal region. Liquid in the geothermal region is an electrically conducting liquid because of high temperature. Hence the study of interaction of the geomagnetic field with the fluid in the geothermal region is of great interest, thus leading to interest in the study of MHD convection flows through porous medium.

The study of heat generation or absorption effects in moving fluids is important in view of several physical problems such as fluids undergoing exothermic or endothermic chemical reactions. The volumetric heat generation has been



assumed to be constant or a function of space variable. Bharathi et al [4] have discussed Non Darcy Hydromagnetic Mixed convective Heat and Mass Transfer flow of a viscous fluid in a vertical Heat generating channel with sources. Balasubramanyam et al [3] have discussed Non Darcy viscous electrically conductive Heat and Mass transfer flow through a porous medium in a vertical channel in the presence of heat generating sources. Hossain et al [15] studied the problem of natural convection flow along a vertical wavy surface with uniform surface temperature in the presence of heat generation or absorption. Hady et al [14] studied the problem of free convection flow along a vertical wavy surface embedded in electrically conducting fluid saturated porous media in the presence of internal heat generation or absorption effect.

Ling and Dybbs [19] has been investigated theoretically a very interesting effect to temperature dependent viscosity on free and mixed convective heat transport in a saturated porous medium. Hossain et al [16], Nasser et al [23] studied the effects of variable properties on natural convection along a vertical wavy surface without porous media. Mallikarjuna [22] have discussed the effect of thermophoresis on convective heat and mass transfer flow over a vertical wavy suface in a porous medium with variable properties.

In this paper, we investigate effect of heat sources on hydromagnetic flow of electricallyt conducting viscous fluid past a vertical wavy surface embedded in a fluid saturated porous medium with thermal radiation with variable properties.By employing the Rungre-Kutta fourth order with Shooting technique, the governing equations have been solved .The effect of thermophoresis, heat sources, radiation, variable viscosity, thermal conductivity on the flow characteristics have been analysed. The obtained results are compared with those presented by Cheng (6) in the absence of heat source(Q=-0),magnetic field(M=0),thermal radiation(Rd=0),thermal

conductivity($\beta=0$),thermophoresis($\tau=0$) and $\theta r \rightarrow \infty$.

 $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

II. FORMULATION OF THE PROBLEM:

We consider a steady incompressible, twodmensional laminar natural convective heat and mass transfer flow over a vertical wavy surface embeddede ina saturated porous medum. The porous medium is uniform and local thermal equilibrium with the fluid. The Darcy law is used to describe the fluid saturated porous medium. The fluid is assumed to be gray, absorbing-emitting radiation but non-scattering medium. The wavy surface profile is given by

$$y = \overline{\sigma}(\overline{x}) = \overline{a}Sin(\frac{\pi\overline{x}}{l})$$
 (2.1)

where l is the characteristic length of wavy surface and \overline{a} is the amplitude of the wavy surface. The wavy surface is maintained at constant temperature Tw which are higher than the ambient fluid temperature T_{∞} . We consider the natural convection-radiation flow in the presence of heat sources to be governed by the following equations under Boussinesq approximations:





(2.2)

$$\frac{\partial}{\partial \overline{y}} (\frac{\mu}{k} \overline{u}) - (\sigma \mu_e^2 H_o^2) \frac{\partial u}{\partial y} = \frac{\partial}{\partial \overline{x}} (\frac{\mu}{k} \overline{v}) + \rho g (\beta_o \frac{\partial T}{\partial y} + \beta_1 T \frac{\partial T}{\partial y} + \beta_c \frac{\partial C}{\partial y})$$
(2.3)



$$\vec{u}\frac{\partial T}{\partial y} + \vec{v}\frac{\partial T}{\partial \vec{y}} = \frac{\partial}{\partial \vec{x}}(\alpha \frac{\partial T}{\partial \vec{x}}) + \frac{\partial}{\partial \vec{y}}(\alpha \frac{\partial T}{\partial \vec{y}}) - \frac{1}{C_p}Q_H(T - T_\infty) - \frac{1}{C_p}\nabla \cdot q_r$$

(2.4)

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_b(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}) - \frac{\partial}{\partial x}(U_T C) - \frac{\partial}{\partial y}(U_T C)$$

The relevant boundary conditions are

$$\vec{u} = 0, \vec{v} = 0, T = T_w, C = C_w at \quad \vec{y} = \overline{\sigma}(\vec{x}) = \overline{a} Sin(\frac{\pi x}{l})$$
$$\vec{u} \to 0, T \to T_{\infty}, C \to C_{\infty} as \quad \vec{y} \to \infty$$
(2.6)

where \overline{u} and \overline{v} are the volume averaged velocity components in the directions of x and y respect tively,T,C are temperature,Concentration respective; ly, ρ is the density of the fluid, μ is the dynamic viscosity of the fluid,k is the permeability of the porous medium, σ is the electrical conductivity, μ_{ρ} is the magnetic permeability, Ho is the strength of the magnetic field,DB is the molecular diffusivity,kc is the coefficient of chemical reaction, β_{a} are the coefficients of thermal expansion, α is the thermal conductivity, q_r is the radiative heat flux ,g is the acceleration due to gravity and $Q_{\rm H}$ is the strength of the heat source.KT is the thermal diffusion ratio, Tm is the mean fluid temperature U_T and V_T are thermophoretic velocities which can be written(seeWu and Greif (36))

$$U_{T} = -\frac{kv}{T_{r}} \frac{\partial T}{\partial x} \quad and \quad V_{T} = -\frac{kv}{T_{r}} \frac{\partial T}{\partial y}$$

$$(2.7)$$

$$k_{nf} = \frac{2C_{s}(\lambda s / \lambda p + C1k_{n})[1 + k_{n}(C1 + C2e^{-C_{3}k_{n})]}}{(1 + 3C_{m}k_{n})(1 + 2\lambda_{g} / \lambda_{p} + 2C_{1}k_{n})}$$

Where C1,C2,C3,Cm,Cs, are constants, λ ,g and λp are thermal conductivities of fluid and diffused particles,respectively and kn is the Knudsen number.

Where k is thermophoteric coefficient which ranges in the values between 0.2 and 1.2 and is defined as (See Talbot et al (33))

By applying Rosseland approximation (Brewester [5]) the radiative heat flux q_r is given by

$$q_r = -\left(\frac{4\sigma^*}{3\beta_R}\right)\frac{\partial}{\partial y} \left[T^{\prime 4}\right]$$

(2.5)

Where σ^* is the Stephan – Boltzmann constant and mean absorption coefficient.

(2.8)

Assuming that the difference in temperature within the flow are such that T'^4 can be expressed as a linear combination of the temperature.We expand T'^4 in Taylor's series about Te as follows

$$T'^{4} = T_{\infty}^{4} + 4T_{0}^{3}(T - T_{0}) + 6T_{0}^{2}(T - T_{0}) + \dots$$
(2.9)

Neglecting higher order terms beyond the first degree in $(T - T_{\infty})$ we have

$$T'^{4} \cong -3T_{0}^{4} + 4T_{0}^{3}T$$

(2.10) Differentiating equation (2.6) with respect to y and using (2.7) we get

$$\frac{\partial(q_R)}{\partial y} = -\frac{16\sigma^* T_0^3}{3\beta_R} \frac{\partial^2 T}{\partial y^2}$$
(2.11)

On using equations(2.11) in the last term of equation(2.4) we get

$$\vec{u} \frac{\partial T}{\partial y} + \vec{v} \frac{\partial T}{\partial \vec{y}} = \frac{\partial}{\partial \vec{x}} (\alpha \frac{\partial T}{\partial \vec{x}}) + \frac{\partial}{\partial \vec{y}} (\alpha \frac{\partial T}{\partial \vec{y}}) - \frac{1}{C_p} Q_H (T - T_\infty) + \frac{16\sigma^{\bullet} T_\infty^3}{C_p \beta_R} (\frac{\partial^2 T}{\partial \vec{x}^2} + \frac{\partial^2 T}{\partial \vec{y}^2}) + Ec(\frac{\partial^2 \psi}{\partial x^{22}} + \frac{\partial^2 \psi}{\partial y^{22}})$$

(2.12)



The fluid properties are assumed to be constant except fluid viscosity and thermal conductivity.Terefore we assume that the viscosity of the fluid is to be an inverse function of the temperature and it can be expressed as[Lai and Kulacki [18]]

$$\frac{1}{\mu} = \frac{1}{\mu_{\infty}} (1 + \delta(T - T_{\infty}) \text{ or } \frac{1}{\mu} = b((T - T_{\infty})$$
(2.13)

Where $b = \frac{\delta}{\mu_{\infty}}$ and $T = T_{\infty} - \frac{1}{\delta}$. Both b and Tr

are constants and their values depend on the reference state and the thermal property of the fluid i.e. δ . In general b>0 for liquids and b<0 for gases , θ_r , which is defined by

$$\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} = -\frac{1}{\delta(T_w - T_\infty)}$$
(2.14)

is constant. The parameter θ_r was first introduced by Ling and Dybbs[19]. It is important to note that $\delta \rightarrow 0$ (*i.e* $\mu = \mu_{\infty} = cons \tan t$) for $\theta_r \rightarrow \infty$, the effect of viscosity is negligible. The value of θ_r is determined by the temperature difference $(T_w - T_\infty)$ and viscosity δ of the fluid in consideration. A smaller values of θ_r is implies either the fluid viscosity changes considerably or the temperature difference is high.On the otherhand, for a larger values of θ_r implies either $(T_w - T_\infty)$ or δ is small, and therefore the effects of variable viscosity can be neglected. In either case the influence of variable viscosity plays very important role and the liquid viscosity varies differently with temperature than that of gases. Therefore, θ_r is positive for gases and negative for liquids respectively.

Also ,we assume that the fluid thermal conductivity α is to be varying as a linear function of temperature in the form [Seddeek and Salem[28]]

 $\alpha = \alpha_o (1 + E(T - T_\infty))$

Where α_o is the thermal diffusivity at the wavy surface temperature Tw and E is a constant depending on the nature of the fluid. It is worth mentioning here that E is positive for fluids such as air and E is negative for fluids such as lubrication oils. This can be written in the nondimensional form [Slattery[30]] as

$$\alpha = \alpha_o (1 + \beta \theta)$$
(2.15)
Where $\beta = E(T - T_{\infty})$ is the thermal conduct

Where $\beta = E(T - T_{\infty})$ is the thermal conductivity parameter. The variation of β can be taken in the range $-0.1 \le \beta \le 0$ for lubrication oils, $0 \le \beta \le 0.12$ for water and

$$0 \le \beta \le 6$$
 for air.

In view of the continuity equation(2.2) we define the stream function ψ as

$$\overline{u} = \frac{\partial \overline{\psi}}{\partial \overline{y}} \quad , v = -\frac{\partial \overline{\psi}}{\partial \overline{x}}$$

(2.16)

In order to write the governing equations in the dimensionless form ,we introduce the following non-dimensional variables as

$$x = \frac{\overline{x}}{l}, y = \frac{\overline{y}}{l}, a = \frac{\overline{a}}{l}, \sigma = \frac{\overline{\sigma}}{l}, \psi^* = \frac{\overline{\psi}}{l}, \theta = \frac{T - T_{\infty}}{T_w - T_{\infty}},$$
$$\phi = \frac{C - C\infty}{C_w - C_{\infty}}$$
(2.17)

The equations (2.12)-(2.15).equations(2.2).(2.3), and (2.10) reduce to

$$(\frac{1}{\theta - \theta_{r}})(\frac{\partial \theta}{\partial y}\frac{\partial \psi^{\bullet}}{\partial y} - \frac{\partial \theta}{\partial x}\frac{\partial \psi^{\bullet}}{\partial x}) + (\frac{\partial^{2}\psi^{\bullet}}{\partial x^{2}} + \frac{\partial^{2}\psi^{\bullet}}{\partial y^{2}}) + Ra(1 - \frac{\theta}{\theta_{r}})(\frac{\partial \theta}{\partial y}(1 + 2\gamma_{l}\theta) + N_{r}\frac{\partial C}{\partial y^{2}}) - M^{2}\frac{\partial^{2}\psi^{\bullet}}{\partial y^{2}}$$

$$(\frac{\partial \theta}{\partial x}\frac{\partial \psi^{\bullet}}{\partial y} - \frac{\partial \theta}{\partial y}\frac{\partial \psi^{\bullet}}{\partial x}) = (1 + \beta\phi + \frac{4Rd}{3})(\frac{\partial^{2}\theta}{\partial x^{2}} + \frac{\partial^{2}\theta}{\partial y^{2}}) + \beta((\frac{\partial \theta}{\partial x})^{2} + (\frac{\partial \theta}{\partial y})^{2}) - Q\theta$$

$$(2.18)$$

$$(2.18)$$

$$(2.18)$$

$$(2.19)$$



$$Le(\frac{\partial\phi}{\partial x}\frac{\partial\psi^{\bullet}}{\partial y} - \frac{\partial\phi}{\partial y}\frac{\partial\psi^{\bullet}}{\partial x}) = (\frac{\partial^{2}\phi}{\partial x^{2}} + \frac{\partial^{2}\phi}{\partial y^{2}}) - Le\gamma\phi - Sc\tau(\frac{\partial^{2}\theta}{\partial x^{2}}\phi + \frac{\partial^{2}\theta}{\partial y^{2}}\phi + \frac{\partial\theta}{\partial x}\frac{\partial\phi}{\partial x} + \frac{\partial\theta}{\partial y}\frac{\partial\phi}{\partial y})$$

(2.20)

Where $Ra = \frac{\beta_T g(T_w - T_{\infty})l}{\alpha_o v}$ is the Darcy-Rayleigh number, $v = \frac{\mu_{\infty}}{\rho}$ is the kinematic viscosity of the

fluid, $Rd = \frac{4\sigma^* T_{\infty}^3}{k_f \beta_R}$ is the Radiation parameter, $Q = \frac{Q_H l^2}{\alpha_o C_p}$ is heat source parameter, $M^2 = \frac{\sigma \mu_e^2 H_0^2 l^2}{\mu}$ is

the magnetic parameter. $Le = \frac{V}{D_B}$ is the Lewis number, $N = \frac{\beta_c (C_w - C_\infty)}{\beta_0 (T_w - T_\infty)}$, $Ec = \frac{\mu (T_w - T_\infty)}{C_p}$ is the Eckert parameter, $\tau = -\frac{k}{T_r} (T_w - T_\infty)$ is the thermophoretic parameter and $Sc = \frac{V}{D_B}$ is the Schmidt

number.

In equation(2.19),

 $\tau = -\frac{k}{T_r}(T_w - T_\infty)$ represents thermophoresis

parameter.From the practical point view two possible cases exist,(i)heated surface $(T_w - T_x > 0)$, which leads to $\tau < 0$, (ii) cold surfaces $(T_w - T_\infty < 0)$, which gives rise τ >0.For the second case thermophoresis is the special type of mechanism for capture of particles on cold surfaces, being especially important for submicron particles since thermophoresis velocity is relatively independent of particle ,while the first one may be thought of suppose the particle deposition on the surface, it concludes that for heated surfaces a dust free stream is obtained adjacent to the surface due to particles moving away from the surface, thus fluid particle concentration is tending to zero near the surface.Hence thermophoretic velocity is treated as particle deposition velocity when $\tau < 0$.

The transformed boundary conditions are

$$\psi^{\bullet} = 0, \theta = 1, \phi = 1 \text{ at } y = aSin(x)$$
$$\frac{\partial \psi^{\bullet}}{\partial y} \to 0, \theta \to 0, \phi \to \infty \text{ as } y \to \infty$$

We can transform the effect of wavy surface from the boundary conditions into the

governing equations by using suitable coordinate transformation with boundary layer scaling, for the case of free convection .The Cartesian coordinates(x,y) are transformed into the new variables(ξ, η).

We incorporate the effect of effect of wavy surface and the usual boundary layer scaling into the governing equations(2.16)&(2.17) for free convection ,using the transformations and $Ra \rightarrow \infty$ (i.e boundary layer approximation),

$$x = \xi, \hat{\eta} = \frac{y - aSin(x)}{\xi^{1/2}Ra^{-1/2}}, \psi^{\bullet} = Ra^{1/2}\psi$$
(2.22)

These transformations are similar to those presented in ,for instance , Rees and Pop[17]. We obtain the following boundary layer equations :



$$(\frac{1}{\theta - \theta_r})(1 + a^2 \cos^2 \xi) \frac{\partial \theta}{\partial \eta} \frac{\partial \psi}{\partial \eta} + (1 + a^2 \cos^2 \xi) \frac{\partial^2 \psi}{\partial \eta^2} = Ra\xi^{1/2}(1 - \frac{\theta}{\theta_r})(\frac{\partial \theta}{\partial \eta} + N\frac{\partial \phi}{\partial \eta}) - M^2 \frac{\partial^2 \psi}{\partial \eta^2}$$

$$\xi^{1/2}\left(\frac{\partial\theta}{\partial\xi}\frac{\partial\psi^{\bullet}}{\partial\eta} - \frac{\partial\theta}{\partial\eta}\frac{\partial\psi^{\bullet}}{\partial\xi}\right) = (1 + a^{1}Cos^{2}(\xi))\left((1 + \beta\phi + \frac{4Rd}{3})(\frac{\partial^{2}\theta}{\partial\eta^{2}}) + \beta(\frac{\partial\theta}{\partial\eta})^{2}\right) - Q\theta$$

(2, 22)

$$\xi^{1/2} Le(\frac{\partial \phi}{\partial \xi} \frac{\partial \psi}{\partial \eta} - \frac{\partial \phi}{\partial \eta} \frac{\partial \psi}{\partial \xi}) = (1 + a^2 Cos^2(\xi))(\frac{\partial^2 \phi}{\partial \eta^2}) - Sc\tau(\frac{\partial^2 \theta}{\partial \xi^2} \phi + \frac{\partial^2 \theta}{\partial \eta^2} \phi + \frac{\partial \theta}{\partial \xi} \frac{\partial \phi}{\partial \xi} + \frac{\partial \theta}{\partial \eta} \frac{\partial \phi}{\partial \eta})$$

$$(2.24)$$

III. SOULUTION METHODOLOGY:

We now introduce the following similarity variables as

$$\eta = \frac{\overline{\eta}}{(1 + a^2 Cos^2(\xi))}, \psi = \xi^{1/2} f(\eta), \theta = \theta(\eta),$$

 $,\phi = \phi(\eta)$

In equations(2.19)&(2.20),we obtain a system of ordinary differential equations as follows:

$$f'' + \left(\frac{1}{\theta - \theta_r}\right)\theta f' - \frac{M^2}{(1 + a^2 \cos^2 \xi)} f'' = Ra(1 - \frac{\theta}{\theta_r}) (\theta h'e + kd)$$
(2.25)
$$\beta(\theta')^2 + \left(1 + \beta\theta + \frac{4Rd}{3}\right)\theta'' + \frac{1}{2}f\theta' - Q\left(1 + a^2 \cos^2(\xi)\right)\theta$$
)
(2.26)
$$Le\phi'' + \left(\frac{1}{2}Lef - Sc\tau\theta'\right)\phi' - Sc\tau\phi\theta'' = 0$$
The h
express number

(2.27) where prime denotes differentiation with respect to η .

The corresponding boundary conditions are

f = 0,
$$\theta$$
=1 , ϕ =1 at η = 0 (2.28)

 $f' \to 0, \theta \to 0, \phi \to 0 \text{ as } \eta \to \infty$ In equation(2.26) the radiation parameter $Rd = \frac{4\sigma^* T_{\infty}^3}{k_f \beta_R}$ means that the rate of thermal radiation contribution relative to the thermal conditions. As $Rd \rightarrow \infty$, influence of thermal radiation is high in the boundary layer regime. For $Rd \rightarrow 0$, the term 4Rd/3 tends to zero. For Rd=1, thermal radiation and thermal conduction will give equal contribution.

The main results of practical interest in many applications are Skin friction coefficient Cf,heat transfer coefficient,mass transfer coefficient at the surface.

)(**D**he kt M_rt d) ion coefficient Cf is given by

$$C_{f} = \frac{f''(0)(1 + a^{2}Cos^{2}(\xi))Ra^{1/2}}{(1 + M^{2} + a^{2}Cos^{2}(\xi))}$$
(2.29)

The heat and mass transfer coefficients are expressed in terms of Nusselt and Sherwood numbers Nux ,Shx.

Nusselt number Nux ans Sherwood number Shx are given by

$$Nux = \frac{xq_w}{\alpha_o(T_w - T_\infty)}, Shx = \frac{xm_w}{D_b(C_w - C_\infty)}$$

(2.24)

Where q_w is the heat flux on the wavy surface, and is defined by



$$q_{w} = -\alpha_{0}\overline{n}.\nabla T \text{ and } \overline{n} = \left(-\frac{aCos(\xi)}{\sqrt{(1+a^{2}Cos^{2}(\xi))}}, \frac{1}{\sqrt{(1+a^{2}Cos^{2}(\xi))}}\right)$$

is the unit normal vector to the wavy surface, α_0 is the effective porous medium thermal conductivity. Therefore

$$Nu_{\xi} = -\frac{\theta'(0)Ra_{\xi}^{1/2}}{\sqrt{(1+a^{2}Cos^{2}(\xi))}} ,$$

$$Sh_{\xi} = -\frac{\phi'(0)Ra_{\xi}^{1/2}}{\sqrt{1+a^{2}Cos^{2}(\xi)}}$$
(2.30)

IV. CONPARISON

Comparison the values of the rate of heat and mass transfer with values reported by Cheng(6) for $B=0,\tau=0.0=0.Rd=0.M=0$ and $\theta r \rightarrow \infty$.

D						
Parame		NuxRa	ShxRa	NuxRa	ShxRa	
ters		-1/2	-1/2	-1/2	1/2	
Le	Ν	Cheng(6)		Present results		
1	4	0.992	0.992	0.9943	0.9944	
10	4	0.681	3.290	0.6815	3.2922	
10	4	0.521	10.521	0.5222	10.520	
0					1	
4	1	0.559	1.358	0.5592	1.3588	
4	2	0.650	1.624	0.6501	1.6244	
4	3	0.728	1.852	0.7282	1.8521	

Comparison the values of the rate of heat and mass transfer with values reported by Mallikarjuna(22) for Q=0,M=0,Rd=0

Parameters			Nux Ra ⁻ 1/2	ShxR a ^{-1/2}	NuxR a ^{-1/2}	Shx Ra ⁻ 1/2
θr	β	τ	Mallikarjuna(22)		Present results	
-	0.	1.	0.58	0.465	0.584	0.46
2.0	5	0	5		9	48
-	0.	1.	0.53	0.434	0.539	0.43
4.0	5	0	4		9	39
-	0.	1.	0.47	0.412	0.474	0.41
10.	5	0	5		8	10
-	1.	1.	0.60	0.494	0.604	0.49
2.0	0	0	5		8	38
-	1.	1.	0.59	0.483	0.595	0.48
2.0	5	0	5		1	36

-	2.	1.	0.59	0.472	0.592	0.47
2.0	0	0	2		1	22
-	0.	1.	0.59	0.395	0.594	0.39
2.0	5	5	5		5	48
-	0.	2.	0.60	0.345	0.604	0.34
2.0	5	0	5		7	49
-	0.	2.	0.61	0.295	0.611	0.29
2.0	5	5	2		2	48

In the absence of mass transfer(N=0) and heat sources(Q=0) and magnetic field(M=0) the results are in good agreement with Mallikarjuna[22]

V. RESULTS AND DISCUSSION:

Figs.2a-2c illustrate the variation of velocity , temperature and concentration with Grashof number(G).It can observed from the profiles that the axial velocity enhances in the flow region with increasing G..An increase in G reduces the temperature and concentration .This may be attributed to the fact that the thickness of the thermal and solutal boundary layers reduce with increasing values of G.

Fig.3a-3c depict the variation of velocity, temperature and concentration with magnetic parameter(M).From the profiles we find that the velocity reduces with G in the entire flow region.The temperature and concentration enhance with increasing values of magnetic parameter.This may be attributed to the fact that the thickness of the thermal and solutal boundary layer decay with increasing M

Figs.5a-5c represent u , θ and ϕ with heat source parameter(Q).From fig.5a, we find that the an increase in the strength of the heat generating source enhances the velocity in the region(0,1.0) adjacent to the wall and reduces far away from the wall,while in the case of heat absorbing source,heat is generated in the region(0,1.0) and the heat energy is absorbed in the region(1.0,4.0).The temperature and the concentration decrease with increase in Q>0 and enhances with increase in Q<0(fig. 5b&5c).

Figs.6a-6c show the variation of velocity and temperature with the influence of radiation parameter(Rd).From fig.6a we find that the velocity reduces in the flow region(0,1.0) and enhances far away from the boundary.This means that the thickness of the momentum boundary layer reduces with increasing values of Rd. Fig.6b&6c represents the temperature and concentration with Rd. It can be seen from the profiles that an increase in Rd leads to thickening of the thermal and solutal boundary layers which results in an enhancement of the temperature and concentration in the flow region.



The variation of non-dimensional velocity and temperature profiles with η for different values of temperature dependent viscosity parameter(θ r) is illustrated in figs.7a-7c.It is found that from fig.7a that the velocity of the fluid reduces in the region(0,2.0) and enhances far away from the wall. This can be explained physically as the parameter θ r increases there is increment in the boundary layer thickness.From fig.7b&7c we notice that the temperature and concentration profiles increase with increasing values of θ r. This can be attributed to the fact the an increase in θ r grows the thickness of the thermal and solutal boundary layers which results in an enhancement of the temperature and concentration in the flow region.

From 8a-8c represent the effect of thermal conductivity parameter β on the non-dimensional velocity, temperature and concentration .Fig.8a shows the variation of velocity with β .In this case the velocity is found to depreciates in the flow region (0,1.0) adjacent to the wall and enhances far away from the wall..From fig.8b we found that as the thermal conductivity parameter β increases the temperature enhances. This is due to the thickening of the thermal boundary layer as a result of increasing values of thermal conductivity. From fig.8c we found that as the thermal conductivity parameter β decreases the concentration reduces in the entire flow region.. This is due to the thinning of the solutal boundary layer as a result of increasing values of thermal conductivity.

The effect of thermophoretic parameter(τ) on u, θ and ϕ can be seen from the figs.9a-9c.From fig.9a we find that increasing the thermophoretic parameter(τ)tends to increase significantly i.e increase the thickness of the momentum boundary layer Fig.9c reveals that the temperature profile depreciate considerably with increase in τ .From fig.9c it is seen that increase in thermophoretic parameter(τ)leads to raise the values of concentration .In otherwords increase in τ decelarates the thermal boundary layer thickness while accelerates the solutal boundary layer thickness.

From figs.10a-10c we notice that an increase in Schmidt number(Sc) increases the velocity in the narrow region(0,0.5) and in the remaining region it reduces .Lesser the molecular diffusivity larger the temperature and smaller the concentration in the flow region(figs.10b&10c)..In otherwords,an increase in Sc decays the momentum boundary layer in (0,0.5) and decays in the region(0.5,4.0).Also the thermal boundary layer

grows and the solutal layer decays with increasing Sc.

Figs.11a-11c represent u, θ and ϕ with amplitude of the wavy surface 'a'.From fig.11a, we find that an increase in amplitude reduces the velocity in the entire flow region.The temperature and concentration increase with increase in 'a' in the entire flow region.

Figs.12a-12c depict the variation of u, $\theta.and \ \varphi$ with streamwise coordinate (ξ).It can be seen from the profiles that an increase in streamwise coordinate increases the velocity and reduces the temperature ,concentration in the flow region.

The skin friction(Cf)at the wall is represented in table.2 for different variations.From the tabular values we find that the magnitude of the skin friction enhances with increase in Ra and M.|C_f| increases at the wall with increase in buovancy ratio(Nr>0) when the buovancy forces are in the same direction and for the forces acting in opposite directions, Cf reduces with Nr<0. Higher the viscosity parameter(θ r) or higher thermal radiation(Rd) or thermophoretic parameter(τ) smaller the skin friction at the wall.An increase in thermal conductivity(β) leads to an enhancement in Cf.Higher the strength of the heat generating/absorbing heat source larger the magnitude of Cf at the wall. Lesser the molecular diffusivity Lewis number (Le) or stream wise coordinate (ξ) larger the skin friction at the wall.Also an increase in amplitude of the wavy surface 'a' leads to a depreciation in the magnitude of C_f at the wall.

The rate of heat transfer (Nu) at the wall is displayed in table.1.From the tabuler values we find that rate of heat transfer at the wall increase with increase in Grashof number(G) or magnetic parameter(M). Increasing buoyancy ratio(Nr>0) enhances the rate of heat transfer(Nu) when the buoyancy forces are in the same directions and for the forces acting in opposite directions we find a reduction in Nu at the wall The Nusselt number reduces with increase in viscosity parameter(θ r). The variation of Nu with heat generating source parameter(Q>0) and reduces with heat absorbing source(Q<0). The Nusselt number reduces with increase in Rd.An increase in thermal conductivity parameter(β) / thermophoretic parameter (τ) increases the Nusselt number at the wall. The rate of heat transfer at the wall grows with increasing Sc.The variation of Nu with amplitude of the wavy surface (a) and streamwise coordinate(ξ) shows that Nu decreases with increase in 'a' and enhances with ' ξ '.



The rate of mass transfer(Sh) at the wall is displayed in table.1.From the tabuler values we find that rate of heat transfer at the wall increase with increase in G or M at the wall.Increasing buoyancy ratio(Nr) enhances Sh when the buoyancy forces are in the same directions and for the forces acting in opposite directions we find a reduction in Sh at the wall. The Sherwood number reduces with increase in viscositv parameter(θ r). The variation of Sh with heat source parameter(Q shows that Sh increases with increase in heat generating/absorbing heat source. The Sherwood number enhances with increase in Rd or thermal conductivity parameter(β). An increase in thermophoretic parameter(τ) depreciates the rate of mass transfer at the wall. The rate of mass transfer Sh at the wall decays with increase in Sc.The variation of Sh with amplitude of the wavy surface (a) and streamwise coordinate(ξ) shows that Sh decreases with increase in 'a' and enhances with 'ξ'.

Paramet		τ(0)	Nu(0)	Sh(0)
er				
R	2	-2.04921	0.647698	0.696629
а	4	-5.68454	0.852137	1.049233
	6	-10.3421	-10.3421 1.00824	
	10	15.4116	1.25569	1.740933
Μ	0.5	-1.67621	0.421996	0.755057
	1.0	-1.75005	0.427222	0.769116
	1.5	-1.92645	0.439276	0.801253
	2.0	3.6587	0.459794	0.855098
Ν	1.0	-0.7438	0.502399	0.445801
r	2.0	-1.57612	0.603067	0.619884
	-0.5	-0.268699	0.356704	0.206066
	-1.5	-0.119499	0.219677	0.031379
Q	0.5	-2.15126	0.513385	0.656064
	1.5	-2.30398	0.566705	0.679954
	-0.5	-1.32029	0.159097	0.662008
	-1.5	-1.39044	0.146142	0.741572
R	0.5	-2.15126	0.513385	0.656064
d	1.5	-2.01417	0.436305	0.694776
	3.5	-1.96932	0.408784	0.706148
	5.0	-1.85545	0.360287	0.710648
S c	0.2 4	-1.71636	0.420728	0.793261
	0.6	-1.67621	0.421996	0.755057

	6			
	1.3 0	-1.57667	0.425086	0.660461
	2.0 1	2.92989	0.428004	0.569004
θ_{r}	-2.0	-0.910404	0.520194	0.476496
,	-4.0	-0.796641	0.508911	0.457001
	-6.0	-0.765534	0.505261	0.450718
	- 10. 0	-0.79865	0.502399	0.445801
β	0.5	-2.15124	0.513385	0.656064
	1.0	-2.17654	0.514721	0.677219
	1.5	-2.18976	0.515567	0.698765
	2.0	-2.224567	0.5178965	0.712345
τ	0.2	-1.67621	0.421996	0.755057
	0.4	-1.60362	0.424257	0.686055
	0.6	-1.53911	0.426232	0.624803
	0.8	2.92951	0.428134	0.564868
a	0.1	-1.51648	0.335793	0.762992
	0.2	-1.51579	0.335749	0.762846
	0.3	-1.51479	0.335686	0.762632
	0.5	2.90888	0.335505	0.762022
ŝ	π	-1.51637	0.335787	0.76297
	$\frac{6}{\pi}$	-1.51648	0.335793	0.762992
	$\frac{4}{\pi}$	-1.51658	0.3358	0.763014
	π	2.91561	0.335806	0.763036





M=0.5,Nr=0.5,Rd=0.5,\beta=0.5,\thetar=-2, Sc=0.24,a=0.2,ξ=π/4 1.0 0.8 0.6 Ra □ 2, 4, 6, 10 0.4 0.2 2 1 3 **Fig.2b** variation of temperature(θ)with Ra M=0.5,Nr=0.5,Rd=0.5,β=0.5,θr=-2, Sc=0.24,a=0.2, \xi=\pi/4 1.0 0.8 0.6 Ra 🗆 2, 4, 6, 10 0.4 0.2 2 3 4 **Fig.2c** variation of Concentration(ϕ) with Ra

 $\begin{array}{l} M{=}0.5, Nr{=}0.5, Rd{=}0.5, \beta{=}0.5, \theta{r}{=}{-}2, \\ Sc{=}0.24, a{=}0.2, \xi{=}\pi/4 \end{array}$















Fig.8a variation of axial velocity(u)with β M=0.5,Nr=0.5,Rd=0.5, θ r=-2.0, Sc=0.24,Ra=2,a=0.2, ξ = $\pi/4$



M=0.5,Nr=0.5,Rd=0.5, β =0.5, θ r=-2.0, Sc=0.24,Ra=2,a=0.2, ξ = $\pi/4$

 $M=0.5,Nr=0.5,Rd=0.5,\beta=0.5,\theta r=-2.0,$ Sc=0.24,a=0.2, $\xi=\pi/4$

$$\label{eq:metric} \begin{split} &M{=}0.5, Nr{=}0.5, Rd{=}0.5, \beta{=}0.5, Sc{=}0.24, \\ &Ra{=}2.0, \theta r{=}{-}2, Ra{=}2, \xi{=}\pi/4 \end{split}$$

Fig.11c variation of Concentration(ϕ)with a M=0.5,Ra=2.0,Rd=0.5, β =0.5,Ra=2, Sc=0.24, θ r=-2.0, ξ = $\pi/4$

M=0.5, Sc=0.24, a=0.2, $\theta r=-2.0$

REFERENCES:

[1]. Alam, M.S, Rahman, M.M and Sattar, M.A: Effect of variable suction and thermophoresis on steady MHD combined free-forced convective heat and mass transfer flow over a semi-infinite permeable inclined plate in the presence of thermal radiation, International Journal of Thermal Sciences, V.47, pp.758-765(2008)

[2]. Aliveli B, Sreevani M : Convective heat and mass transfer flow of viscous fluid with variable viscosity in the presence of thermophoresis particle deposition, International Journal for Research &

Development in Technology, Vol.8, Issue 5 (2017), Issn 2349-3585, web.www.ijrdt.org.

[3]. Balasubramanyam S, Int. Journal of Appl. Math & Mechanic, Vol 6 (15) pp.33-45, (2010).

- [4]. Bharathi. M, Suresh.M, Rajeswara Rao U and Prasada Rao DRV, Journal of Pure and Appl. Physics. Vol 21, No.2, pp. 205-216, (2009).
- [5]. Brewester, M.Q: Thermal radiative transfer and properties. John Wiley & Sons. Inc. New York (1992).
- [6]. Cheng, C.L and Chan, K.L: Combined effects of thermophoresis and electrophoresis on particle deposition onto a wavy surface disk, International Journal Heat and Mass transfer, V.51, pp.2657-2664(2008)
- [7]. Damseh R.A., Tahat M.S, and Benim A.C, Non-similar solutions of magneto hydrodynamic and thermophoresis particle deposition on mixed convection problem in porous media along a vertical surface with variable wall temperature, Progress in Computational Fluid Dynamics 9(1), 58-65, (2009).
- [8]. Das, K: Influence of thermophoresis and chemical reaction on MHD micro polar fluid flow with variable fluid properties, International journal of heat and Mass transfer,V.55,pp.7166-7174(2012)
- [9]. Dinesh K.K. and jayaraj S, Augmentation of thermophoretic deposition in natural convection flow through a parallel plate channel with heat sources, Int. Communications Heat and Mass Transfer 36, 931-935 (2009).
- [10]. Duwairi H.M., Damseh R.A., Effect of thermophoresis particle deposition on mixed convection from vertical surfaces embedded in saturated porous medium, Int. J.Numerical Methods Heat Fluid Flow, 18(2), 202-216 (2008)
- [11]. Ganesan,P,Suganthi,R.K,Loganathqam,P:M echanica,V.102,pp.125-132(2013)
- [12]. GorenS.L: Thermophoresis of aerosol particles in the laminar boundary layer on a flat plate, Journal of Colloid Interface ScienceV.61pp.77-85(1977)
- [13]. Grosan T, Pop R, and Pop I, Thermophoretic deposition of particles in fully developed mixed convection flow in parallel plate vertical channel, Heat Mass Transfer 45, 503-509, (2009).
- [14]. Hady F.M., Mohamed R.A., Mahdy A.: MHD free convective flow along a vertical

wavy surface with heat generation or absorption effect Int. Comm. Heat Mass transfer, 33 (2006).

- [15]. Hossain M.A., Molla M.M., Yaa L.S.: Natural convective flow along a vertical wavy surface temperature in the presence of heat generation/ absorption, Int. J. Thermal Science 43, pp157-163 (2004).
- [16] Hossain, M.A, Kabir, S and Rees, D.A.S: Natural convection of fluid with temperature dependent viscosity from heated vertical wavy surfaceZ.Angew.Math.phys.V.53,pp.48-52(2002)
- [17]. Jayaraj C Dinesh K and Pillai K.L: Thermophoresis in natural convection with Variable properties, Heat and Mass transfer V.34pp.469-475(1999)
- [18] LaiF.C and Kulacki,F.A: Free and Mixed Convection from Slender Bodies of Revolution, International Journal of Heat and Mass transfer, V.33 (5), pp.1028-1031(1990).
- [19]. Ling, J.X and Dybbs, A: The effect of variable viscosity on forced convection over a flat plate submerged in a porous medium, ASME Journal of Heat transfer,V.114,pp.1063-065(1992).
- [20]. Liu Z, Chen Z, and Shi M: Thermophoresis of particles in aqueous solution in micro channel, Applied Thermal Engineering, 29, (5-6), 1020-1025, (2009).
- [21]. Mahdy A, Hady f.M.: Effect of thermophoretic particle deposition in non-Newtonian free convection flow over a vertical plate with magnetic field effect, Journal of Non-Newtonian fluid Mechanics, 161 (1-3), 37-41 (2009).
- [22]. Mallikarjuna B : Convective heat and mass transfer in viscous fluid flow over a porous medium, Ph.D. thesis, JNTUA, Ananthapuramu, June 2014.
- [23]. Nasser, S.E and Nader Y.A.E: The effects of variable properties on MHD unsteady natural convection heat and mass transfer over a vertical way surface, MecccanicaV.44pp.573-586(2009)
- [24]. Postelnicu A, Effects of thermophoresis particle deposition in free convection boundary layer from a horizontal flat plate embedded in a porous medium, Int. J. Heat Mass Transfer 50 (15-16), 2981-2985 (2007).
- [25]. Postelnicu,A: Thermophersis partical deposition in natural convection over inclined surfaces in porous media,

International journal oh Heat and Mass transfer.V.55,pp.2087-2094(2012)

- [26]. Puvi Arasu, P, Loganatham, P, Kandaswamy, R and Muhaimin, I: Nonlinear Analysis; hybrid Systems, V.5, pp. 20-31(2011)
- [27]. Seddeek, M.A : Finite-element method for the effects of chemical reaction, variable viscosity, thermophoresis and heat generation/absorption on a boundary-layer hydromagnetic flow with heat and mass transfer over a heat surface, Acta Mechanica,V.177,pp.1-18(2005)
- [28]. Seddeek, M.S and Salem, A. M: Laminar mixed convection adjacent to vertical continuously stretching sheets with variable viscosity and variable thermal diffusivity, Heat and Mass transfer, V.41, pp.1048-1055(2005)
- [29]. Selim, A, Hossain, M.A and Ress, D.A.S: The effect of surface mass transfer on mixed convection flow past a heated vertical flat permeable plate with thermophoresis, International Journal of Thermal Sciences.V.42, pp.973-982(2003)
- [30]. Slattery, J.C : Momentum Energy and mass transfer in continua, Mc.Graw-Hill, New York(1972)
- [31]. Sreenivasa Reddy B, Madhusudhana K and Sreedhar Babu M : The effect of thermo

porous on convective heat and mass transfer flow past stretching sheet with Soret and Dufour effect, International Research Journal of Mathematics, Engineering and I.T., Vol.3(6), (2015).

- [32]. Sreenivasa Reddy B: Soret and Dufour effect on convective heat and mass transfer flow of a micro polar fluid in presence of thermo porous deposition particle, National Conference on Emerging Research Trends in Pure and Applied Mathematics, S.P. Mahila University, Tirupati,(2017)
- [33]. Talbot,L ,Cheng,R.K, Schefer, R.W and Willis, D.R: Thermophoresis of particles in a heated boundary layer, Journal of Fluid Mechanics,V.101,pp.737-758(1980)
- [34]. Tsai,R and Huang,J.S :Combined effects of thermophoresis and electrophoresis on particle deposition onto a vertical flat plate from mixed convection flow through a porous medium, Chemical Engineering Journal,V.157,pp.52-59(2010)
- [35]. Wang,C.C and Chen,C.K: Acta Mechanica,V.181,pp.139-151(2006)
- [36]. Wu,C.K and Grief,R: Thermophoretic deposition including an application to the outside vapor deposition process, International Journal of heat and Mass transferV.39 (7) pp.1429-1438(1996)